**A geometric approach to blob detection in color images**

**(A Riemahniann approach to blob detection in color images)**

**Abstract**: Full use of color in keypoints detection is an important problem in computer vision. (Full use of color information is very important for quality of keypoints detection) (Full use of color has a big importance for keypoints detection. ) Most methods of keypoints detection methods in color images use local or global conversion of image to grayscale (utilize in some way a conversion of image to grayscale). By conversion features become less distinguishable and their ordering based on their strength is being distorted.

In this paper we propose the generalization for color images of blob detection, a widely used keypoints detection method. We consider image surface as a section of trivial vector bundle over smooth manifold and reformulate original method through the (mean and scalar ) this section. This formulation provides a straightforward and (natural) generalization for color images. Different computational strategies are presented. This makes our method applicable to a broad range of problems and data models. The effectiveness (and superiority) of the proposed method is shown by the experiments of our method application for task of chemical compounds classification. The algorithm is easy to implement and as fast as original method.

(Blob detection is a classical low-level computer vision method. Informally speaking, it aims to find circle-like regions of different sizes in grayscale image with similar intensity inside. It is the base of widely used SIFT and SURF key-point detection methods. Recently generalizations for grayscale images on surfaces were proposed. But the generalization for color images is not so straightforward. A naïve approach of per-channel blob detection has serious drawbacks. Existing more sophisticated generalization for color images can be described as converting image to grayscale in some way and the apply base method.

In this paper we propose a generalization of blob detection for color images. It is based on reformulation of original method through the curvatures of image plot. Tests on chemical classification dataset show the superiority of the proposed approach. The theoretical results include a formulation of curvature-based blob detection functions through hessian invariants, volumes of plot and base manifold, proofs of preservation of the original method properties and connection with previously proposed generalizations. Also calculations through curvature techniques are proposed. )

**Introduction**

Keypoints detection is applied in a wide range of tasks - image mosaicing, image registration, panorama stitching, 3D recovery, tracking, recognition, shapes modeling, etc. Its usage is not limited to planar images, it is used also for 2d-surfaces[], volumetric data[], video data[], etc. Keypoints are usually (seeked) found as low-level features, likely to be stable under some transformations.

Blob detection [, , , , ,] is a widely used keypoints detection method. Informally speaking, it aims to find ellipse-like regions of different sizes in grayscale image with “similar” intensity inside. It is the base of widely used SIFT [] and SURF [] features extraction methods. It can be applied not only to images on Euclidian domain, but also to scalar functions on manifolds[, , ,]. It has applications for different problems with different kinds of data - 3D face recognition, object recognition, panorama stitching, 3D scene modeling[], tracking[], action recognition[], medical images processing[].

For color images full use of color information is important for quality of keypoints detection. Without full use of color vectors features can become less distinguishable from background, also features ordering can be distorted. This can lead to unstable keypoints detection and matching. As to our knowledge all previously proposed generalizations of blob detection for color images are based on global [,,,] or local [,,,] conversion to grayscale. We propose another approach for color images blob detection. It is based on reformulation of original method through the (mean and scalar) curvatures of image surface. This formulation provides a straightforward and (natural) generalization for color image.

Establishing a connection between image processing methods and the geometry of image surface is of its own interest. It helps deeper understand traditional methods[,], provides unexpected insights[,] and gives natural extensions of classical methods to color images[,]. The connection between blob detection and image surface curvatures arose in several papers [, ,], but there were errors in proposed analysis. So our paper is the first to accurately analyze this question in the general setting.

Recently there is a big demand in processing data (naturally) lying on manifolds. In informational geometry [, ,], proteins modeling [, ,], chemical compounds classification[, ,], 3d reconstruction [,], 3d models recognition [], action recognition we deal with data, placed on the manifolds. Because of this, methods, formulated in a general setting, are needed. All previously published color blob detection approaches were defined in a narrow setting – for image in 2D Euclidian domain. We consider image as a smooth vector-valued function over n-dimensional manifold. Also we provide relation of our method to different characteristics of image surface (different computational strategies). So our method is applicable to a broad set of problems and data representations.

It is important to notice, that speed of computation is a crucial aspect for many image processing applications. A large amount of research was devoted to make blob detection faster [, , ,] by using different approximations, sampling strategies, etc. Our method is easily implemented on the base of original blob detection, so every fast implementation of blob detection can be extended to obtain the implementation of our method.

**Contributions:**

We present a generalization of blob detection method for color images, defined as vector-valued function over smooth n-dimensional manifold, based on reformulation of original method through (mean and scalar) curvatures of vector bundle section defined by image.

1. We are the first, to our knowledge, to present blob detection for color images without usage of global or local conversion to grayscale. The algorithm is as fast as grayscale blob detection and easy to implement on the base of original method.
2. We are the first to our knowledge to establish (find out, prove, analyze) a connection between blob detection method and (mean and scalar) curvatures of image surface, which is considered section of trivial vector bundle over smooth manifold.
3. We are the first to analyze color blob detection for a general case of a m-dimensional vector function over smooth n-dimensional manifold. Different computational strategies (formulations) are presented. So our method applicable for a broad set of problems and data representations.
4. We provide experimental results in application of our method to the task of chemical compounds classification. The results show the effectiveness (and superiority) of the proposed approach.

The remainder of the paper is organized as follows. In section 2 we review related work (blob detection algorithms, color blob detection methods, also image processing algorithms related to geometry of image surface). In section 3 we present our main result – color blob detection through image surface (mean and scalar) curvatures (with theoretical analysis). In section 4 we show how to compute our method (how to implement our method). In section 5 we present relation of our method to the different surface characteristics. In section 6 we propose the results of application of our method to the task of chemical compounds classification. In section 7 we give conclusions and discuss possible directions of the future research.

**Related work**

**Blob detection**

Blob detection was firstly proposed in works [, , , ,] for images on 2D Euclidian domain. Different versions of this method were used as a part of well-known SIFT [] and SURF [] algorithms. In works [, ,] blob detection was generalized (in different ways) for finding keypoints locations and sizes on 2D surfaces embedded in and represented by triangular mesh. The theoretical analysis of blob detection and linear scale-space for images of Euclidian domain was given in [, ,], and for 2D surfaces – in [,].

**Color blob detection**

Some approaches were proposed to adapt blob detection for color images on 2D Euclidian domain. These approaches are based on global or local conversion of image to grayscale. In [] processed image is converted to grayscale by projecting color onto vector, obtained by applying PCA to color vectors. In [] authors propose local projection on vector function obtained by applying Laplacian to each image channel. In [] authors propose local projection on vector function which is found by maximizing “blobness” of projection.

**Image processing by (based on) image surface geometry**

Geometry of image surface was used in different image processing methods. In [] metric of image surface was used for generalization of gradient detection to color images. In [, ,] the framework was proposed which allows to reformulate many diffusion methods as Polyakov action on image manifold. In [, ,] image was considered as a purely discrete object – discrete cell complex. The applications of discrete Forman curvatures to image processing were presented. In [, ,, ] image was considered as a section of trivial Clifford bundle over smooth manifold which allows for generalized definitions of gradient, diffusion, Fourier transform.

**Proposed method**

**Grayscale blob detection**

We will list here the formal definition of the general method for the surfaces:

Suppose we have . Blob detection method is as following:

1. Calculate the scale-space: the scaled solution of heat equation on surface

, where is a Laplace-Beltrami operator on surface.

1. Rescale
2. Calculate feature response or where is a Hessian of as a function of with fixed .
3. Find blobs centers as . Find blobs radius as ;

**Color blob detection**

Let’s consider the case of vector-valued image . The generalization of steps 1 and 2 (scale-space construction) to the color images is straightforward because of the linearity of the heat equation. Then look at the step 3. For the vector-valued function its Hessian is a covariant differential of the function differential [] , so we can’t use и as feature response functions.

How can this problem be solved without conversion of to scalar function? (The key observations are the following:

1. If tangent plane of grayscale image plot is “close” to the plane then mean and Gaussian curvatures of image plot are close to the trace and determinant of image Hessian.
2. The cases of color and grayscale images differ in terms of image plot only by its co-dimension in embedding of it to .
3. The mean and Gaussian curvatures for 2D surfaces are only special cases of mean curvature vector norm [] and scalar curvature[]. These curvatures are defined for all co-dimensions)

**Definitions**

Consider a smooth connected manifold (isometricaly embedded in ). Denote metric on it and associated Levi-Civita connection.

There is a vector-valued function . Denote a trivial vector bundle with base manifold . is a smooth (n+m)-dimensional manifold.

Let be a section of . Let be an image of , i.e. . is a smooth n-dimensional manifold, embedded in by definition.

Let . is embedded in by .

Our main result follows from the following theorems:

**Theorem 1**:

,

where is a scalar curvature of a manifold in point p, is a mean curvature vector of a manifold embedded in .

As mean and scalar curvatures are defined for all dimensions and co-dimensions Theorem 1 gives an immediate feature response function for color images. Theorem 2 provides a usual view of feature response function through the invariants of the Hessian.

**Theorem 2**:

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where ,